1. State & Verify Laws by using :-

1. Properties of 0 and 1:
   
   \[0 + X = X, \quad 1 + X = 1, \quad 0 \cdot X = 0, \quad 1 \cdot X = X\]

2. Idempotency Law:
   
   (a) \(X + X = X\)  (b) \(X \cdot X = X\)

3. Involution Law:
   
   \[\overline{\overline{A}} = A\]

4. Complementary Law:
   
   (a) \(X + \overline{X} = 1\)  (b) \(X \cdot \overline{X} = 0\)

5. Commutative Law:
   
   (a) \(X + Y = Y + X\)
   (b) \(X \cdot Y = Y \cdot X\)

6. Associative Law:
   
   (a) \(X + (Y + Z) = (X + Y) + Z\)
   (b) \(X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z\)

7. Distributive Law:
   
   (a) \(X(Y + Z) = XY + XZ\)
   (b) \(X + YZ = (X + Y)(X + Z)\)

8. Absorption Law:
   
   (a) \(X + XY = X\)
   (b) \(X(X + Y) = X\)

9. Third Distributive Law:

   \[X + \overline{X} \cdot Y = X + Y\]

10. De Morgan's Theorems

   (1) \(\overline{X + Y} = \overline{X} \cdot \overline{Y}\)
   (2) \(\overline{X \cdot Y} = \overline{X} + \overline{Y}\)

1. State and algebraically verify Absorption Laws.

   Ans:

   Absorption law states that (i) \(X + XY = X\) and (ii) \(X(X + Y) = X\)

   (i) \(X + XY = X\)
   
   \[\text{LHS} = X + XY = X(1 + Y)\]
   
   \[= X \cdot 1 [\because 1 + Y = 1] \]
   
   \[= X = \text{RHS. Hence proved.}\]

   (ii) \(X(X + Y) = X\)
   
   \[\text{LHS} = X(X + Y) = X \cdot X + XY\]
   
   \[= X + XY\]
   
   \[= X(1 + Y)\]
   
   \[= X \cdot 1\]
   
   \[= X = \text{RHS. Hence proved. [I mark for the statement]}\]

   [I mark for proving it algebraically]
2. State and verify Distributive Laws algebraically.

Distributive law state that (a) \( X(Y + Z) = XY + XZ \)
(b) \( X + YZ = (X + Y)(X + Z) \)

now proof for 1st no. is as simple as we can see

\[
= XY + XZ \\
= X(Y + Z)
\]

L.H.S=R.H.S.

now proof for 2nd. law

\[
\text{R.H.S.} = (X + Y)(X + Z) \\
= XX + XZ + XY + YZ \quad (XX = X \text{ Indempotence law}) \\
= X + XY + XZ + YZ = X(1 + Y) + Z(X + Y) \\
= X.1 + Z(X + Y) \quad (1 + Y = 1 \text{ property of 0 and 1}) \\
= X + XZ + YZ \quad (X \cdot 1 = X \text{ property of 0 and 1}) \\
= X(1 + Z) + YZ \quad \text{R.H.S. Hence proved.}
\]

3. State and verify Demorgan’s Laws algebraically.

By Complementarity Law,

\[
P + \overline{P} = 1 \quad \text{and} \quad P \cdot \overline{P} = 0
\]

(Note: I shall only be using \( P + \overline{P} = 1 \) as its dual is automatically true)

First Law: DeMorgan’s 1st law states \( \overline{X + Y} = \overline{X} \cdot \overline{Y} \)

It is sufficient to prove that \( (X + Y) + \overline{X} \cdot \overline{Y} = 1 \)

\[
\text{LHS} = Y + (X + \overline{X} \cdot \overline{Y}) \\
= Y + X + \overline{Y} \\
= (Y + \overline{Y}) + X \\
= 1 + X \\
= 1 = \text{RHS}
\]

Second Law: DeMorgan’s 2nd Law states that \( \overline{X \cdot Y} = \overline{X} + \overline{Y} \)

It is sufficient to prove that \( X \cdot Y + (\overline{X} + \overline{Y}) = 1 \)

\[
\text{LHS} = \overline{Y} + (\overline{X} + \overline{X} \cdot Y) \\
= \overline{Y} + (\overline{X} + Y) \\
= (Y + \overline{Y}) + \overline{X} \\
= 1 + X \\
= 1 = \text{RHS}
\]

4. Prove algebraically the third distributive law \( X + X'Y = X + Y \).

L.H.S. = \( X + X'Y \)

\[
= X.1 + X'Y \quad (X \cdot 1 = X \text{ property of 0 and 1}) \\
= X(1 + Y) + X'Y \quad (1 + Y = 1 \text{ property of 0 and 1}) \\
= X + XY + X'Y \\
= X + Y(X + X') \\
= X + Y.1 \quad (X + X' = 1 \text{ complementarity law}) \\
= X + Y \quad (Y \cdot 1 = Y \text{ property of 0 and 1}) \\
= \text{R.H.S.} \quad \text{Hence proved.}
\]
2. Verify the following algebraically:- (2)

1. \((A'+B').(A +B)=A'.B+A.B'\)

   Ans:
   
   \[
   \text{LHS} \quad (A' + B') \cdot (A + B) \\
   = A' \cdot A + A' \cdot B + A \cdot B' + B' \cdot B \\
   = 0 + A' \cdot B + A \cdot B' + 0 \\
   = A' \cdot B + A \cdot B' \\
   = \text{RHS (Verified)}
   \]

2. Verify the following using Boolean Laws.

   \(X + Y' = X \cdot Y + X \cdot Y' + X'.Y'\)

   Ans:
   
   \[
   \text{L.H.S} \\
   = X + Y' \\
   = X \cdot (Y + Y') + (X + X') \cdot Y' \\
   = X \cdot Y + X \cdot Y' + X \cdot Y' + X' \cdot Y' \\
   = X \cdot Y + X \cdot Y' + X' \cdot Y' \\
   = \text{R.H.S}
   \]
   
   OR
   
   \[
   \text{R.H.S} \\
   = X \cdot Y + X \cdot Y' + X' \cdot Y' \\
   = X \cdot (Y + Y') + X' \cdot Y' \\
   = X \cdot 1 + X' \cdot Y' \\
   = X + X' \cdot Y' \\
   = X + Y' \\
   = \text{L.H.S}
   \]

3. 

4. Verify the following using Truth Table. (2)

   1. \(U \cdot (U' + V) = (U + V)\)
   
   2. \(X+Y. Z=(X+Y).(X+Z)\)
   
   3. \(X+ (Y+Z) = (X+Y) + Z\)
   
   4. \((A+B+C+D) = A\)
   
   5. \(A + A' = A + B'\)
   
   6. \((x + y + z)(x' + y + z) = y + z\)
   
   7. \(A'B'C + A'BC + AB'C = A' + B'\)

5. Name the law shown below and verify it using a truth table. (2)

   1. \(A+B.C = (A+B).(A+C) -> \text{Distributive Law}\)
   
   2. \(X+X'.Y=X+Y -> \text{third distributive law}\)
6. What does duality principle state? What is its usage in Boolean algebra?

The principle of duality states that starting with a Boolean relation, another Boolean relation can be derived by:
1. Changing each OR sign (+) to an AND sign (·).
2. Changing each AND sign (·) to an OR sign (+).
3. Replacing each 0 by 1 and each 1 by 0.
Principle of duality is used in Boolean algebra to complement the Boolean expression.

7. Write dual of the following Boolean Expression:
   (a) \((x + y')\) (b) \(xy' + x'y + x'\) (c) \(a + a'b + b'\) (d) \((x + y' + z)(x + y)\)
   Ans:
   (a) \(xy'\) (b) \((x + y)(x + y')(x' + y)\)
   (c) \(a . (a' + b) . b'\) (d) \(xy'z + xy\)

8. Write the equivalent expression for the following Logical Circuit: (2)
9. Draw a logical Circuit Diagram for the following Boolean Expression: \( 2 \)

1. \((U + V').W' + Z\)

\begin{align*}
\text{Ans } &
\begin{array}{c|c}
\text{U} & \text{V} \\
\bullet & \bullet \\
\hline
\text{W} & \text{W'} \\
\text{Z} & \text{Z} \\
\end{array}
\end{align*}

\begin{align*}
\text{Ans: } &
\begin{array}{|c|c|c|}
\hline
\text{U} & \text{V} & \text{W'} \\
\text{V} & \text{V'} & \\
\text{W} & \text{W'} & \\
\hline
\text{U} & \text{V} & \text{W'} & \text{Z} \\
\end{array}
\end{align*}

2. Draw the logic circuit for \( F = AB' + CD' \)

Ans:

\begin{align*}
\text{Ans: } &
\begin{array}{|c|c|c|c|}
\text{A} & \text{B} & \\
\text{B} & \\
\text{C} & \\
\text{D} & \\
\hline
\text{A} & \text{B} & \text{D} & \text{D'} \\
\end{array}
\end{align*}

10. Draw a logical Circuit Diagram for the following Boolean Expression by using NOR gate and NAND Gate

\begin{align*}
\text{Solution: } &
\begin{array}{|c|c|c|}
\hline
\text{X} & \text{Y} & \text{Z} \\
\text{Y'} & \text{Z} & \\
\text{Z} & \text{X} & \text{X'} \\
\hline
\text{X} & \text{Y} & \text{Z} & \text{Z'} \\
\end{array}
\end{align*}

\begin{align*}
\text{Solution: } &
\begin{array}{|c|c|c|}
\hline
\text{X} & \text{Z} & \text{Z'} \\
\text{Y} & \\
\text{Z} & \text{Z'} \\
\hline
\text{X} & \text{Z} & \text{Z'} & \text{Z'} \\
\end{array}
\end{align*}
11. CANONICAL SOP AND POS

1. What do you mean by canonical form of a Boolean expression? Which of the following are canonical?
   (i) \(ab + bc\) (ii) \(abc + a'bc + ab'c'\) (iii) \((a + b)(a' + b)\)
   (iv) \((a + b + c)(a + b' + c)(a' + b + c')\) (v) \(ab + bc + ca\)

Boolean Expression composed entirely either of Minterms or maxterms is referred to as canonical form of a
Boolean expression.
(i) Non canonical (ii) canonical (iii) canonical (iv) canonical (v) Non canonical
Type 1:

Q: 1.(i) Express P+Q'R in canonical SOP form. (1)

(ii) \( X + X'Y + X'Z' \)
\[ = X(Y + Y') + X'Y(Z + Z') + X'Z'(Y + Y') \]
\[ = (XY + XY') + X'YZ + X'YZ' + X'Y'Z' \]
\[ = Z(XY + XY') + X'YZ + X'Y'Z' \]
By removing duplicate terms we get the canonical Sum-of-Product form:
\[ XYZ + XY'Z + XYZ' + X'Y'Z' + X'Y'Z' \]

Q: 2.(i) Express P+Q'R in POS form. (1)

Ans::

\[ P + Q'R = (P + Q')(P + R) \]
[By Distributive Law]
\[ = (P + Q' + RR')(P + R + QQ') \]
\[ = (P + Q' + R)(P + Q' + R')(P + R + Q)(P + R + Q) \]  [By Removing the duplicate terms]

(ii) \( (X + Y)(Y + Z)(X + Z) \)
\[ = (X + Y + ZZ')(X + Y + Z)(X + YY' + Z) \]
By removing duplicate terms, we get the canonical Product-of-Sum form:
\[ (X + Y + Z)(X + Y + Z')(X' + Y + Z)(X + Y' + Z) \]

Type 2:

Q: 3

Given the truth table of a function \( f(x, y, z) \), write S-O-P and P-O-S expression from the following truth table:

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>0</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Add a new column containing Minterms and Maxterms. Now the table is as follows:

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>F</th>
<th>Minterms</th>
<th>Maxterms</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( XY'Z' )</td>
<td>( X + Y + Z )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>( XY'Z' )</td>
<td>( X + Y + Z )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>( XY'Z' )</td>
<td>( X + Y + Z )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( XY'Z' )</td>
<td>( X + Y + Z )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>( X'Y'Z' )</td>
<td>( X + Y + Z )</td>
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<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>( X'Y'Z' )</td>
<td>( X + Y + Z )</td>
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<td>1</td>
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<td>0</td>
<td>1</td>
<td>( X'Y'Z' )</td>
<td>( X + Y + Z )</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( X'Y'Z' )</td>
<td>( X + Y + Z )</td>
</tr>
</tbody>
</table>

Now by adding all the minterms for which output is 1, we get the desired sum-of-products expression which is:
\[ XYZ + XY'Z + XYZ \]
Now by multiplying Maxterms for the output 1s, we get the desired product of sums expression which is:
\[ (X + Y + Z)(X + Y + Z')(X + Y' + Z')(X + Y + Z) \]
Type 3:

**Q: 4 Convert the following Boolean expression into its equivalent Canonical Product of Sum form:**

\[X.Y'.Z + X'.Y.Z + X'.Y.Z'\] \hspace{1cm} (1)

<table>
<thead>
<tr>
<th>(X)</th>
<th>(Y)</th>
<th>(Z)</th>
<th>(F)</th>
<th>(\text{POS})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(X + Y + Z)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>(X + Y + Z')</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>(X' + Y + Z)</td>
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<td>(X' + Y + Z)</td>
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<td>(X' + Y' + Z)</td>
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<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>(X' + Y' + Z')</td>
</tr>
</tbody>
</table>

\[F(X,Y,Z) = (X + Y + Z)(X + Y + Z')(X' + Y + Z)(X' + Y' + Z)(X' + Y' + Z')\]

Type 4:

**Q.5**

---

12. **K-MAP**

**Following each question carry 3 marks:**

1)
1) Reduce the following Boolean Expression using K-Map:
\[ F(A, B, C, D) = \Sigma(0, 1, 2, 4, 5, 6, 8, 10) \]

2) Reduce the following Boolean Expression using K-Map:
\[ F(U, V, W, Z) = \Pi(0, 1, 2, 4, 5, 6, 8, 10) \]
3) Reduce the following Boolean Expression using K-Map:
\[ F(A,B,C,D) = \pi(0,1,2,4,5,6,8,10) \]

4) Reduce the following Boolean Expression using K-Map:
\[ F(X, Y, Z, W) = \Sigma(0,1,3,4,5,7,9,10,11,13,15) \]

5) Reduce the following Boolean expression using K map:
\[ F(M,N,O,P) = \Sigma(0,1,3,4,5,6,7,9,10,11,13,15) \]

6) Obtain simplified form for a Boolean expression
\[ F(x,y,z,w) = \Sigma(1,3,4,5,7,9,11,12,13,15) \] using K Map

7) \[ F(P,Q,R,S) = \Sigma(0,3,5,6,7,11,12,15) \]

8) \[ F(P, Q, R, S) = \Sigma(3,5,7,10,11,13,15) \]

9) \[ F(X,Y,Z,W) = \Sigma(0,3,4,5,7,11,13,15) \]

10) \[ F(P,Q,R,S) = \Pi(0,3,5,6,7,11,12,15) \]

11) \[ F(P, Q, R, S) = \Sigma(1,2,3,5,6,7,9,11,12,13,15) \]

12) \[ F(A,B,C,D) = \Sigma(0,1,3,4,5,7,8,10,12,14,15) \]

13) \[ F(A,B,C,D) = \Pi(5,6,7,8,9,12,13,14,15) \]

14) \[ F(X,Y,Z,W) = \Sigma(0,1,4,5,7,8,9,12,13,15) \]

15) \[ F(A,B,C,D) = \Sigma(0,2,3,4,6,7,8,10,12) \]

16) \[ F(U,V,W,Z) = \pi(0,1,2,4,5,6,8,10) \]

17) \[ F(P,Q,R,S) = \Sigma(3,5,8,11,12,15) \]

18) \[ F(U,V,W,Z) = \Sigma(0,1,2,3,4,10,11) \]

19) \[ F(P, Q, R, S) = \Sigma(0,1,2,4,5,6,8,12) \]

20) \[ F(A, B, C, D) = \Sigma(1,3,4,5,6,7,12,13) \]

21) \[ F(U, V, W, Z) = \Pi(0,1,3,5,6,7,15) \]

22) \[ F(X, Y, Z, W) = \Sigma(0,1,6,8,9,10,11,12,15) \]

\[ \text{Ans:} \]

\[ \text{Simplified Expression: } XY' + Y'Z' + XZ'W' + XZW + X'YZW' \]

(½ Mark for each of grouping - 5 groups x ½ = 2½ Marks)
(½ Mark for writing final expression in reduced/minimal/non redundant form as \( XY' + Y'Z' + XZ'W' + XZW + X'YZW' \))

Note: Deduct ½ mark if wrong variable names are used